# REVIEW – QUADRATIC FUNCTIONS

11U

1. Solve the following. All answers must be exact and simplified.

a. $5x^2 - 4x - 20 = 0$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(5)(-20)}}{2(5)}$$

$$x = \frac{4 \pm \sqrt{416}}{10} = \frac{2 \pm 2\sqrt{26}}{5}$$

b. $3x^2 - 5x + 2 = 0$

$$(x-1)(3x-2) = 0$$

$$\therefore x = 1 \text{ or } x = \frac{2}{3}$$

c. $2(x-2)(x+1) - (x+3) = 0$

$$2(x^2 - x - 2) - x - 3 = 0$$

$$2x^2 - 2x - 4 - x - 3 = 0$$

$$2x^2 - 3x - 7 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 + 56}}{4}$$

$$x = \frac{3 \pm \sqrt{65}}{4}$$

d. $3(x-1)(x+4) - 2(2x+1)^2 = -18$

$$3(x^2 + 3x - 4) - 2(4x^2 + 4x + 1) + 18 = 0$$

$$3x^2 + 9x - 12 - 8x^2 - 8x - 2 + 18 = 0$$

$$-5x^2 + x + 4 = 0$$

$$5x^2 - x - 4 = 0$$

$$(x-1)(5x+4) = 0$$

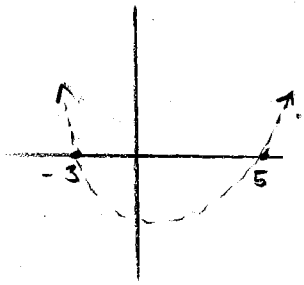
$$\therefore x = 1 \text{ or } x = -\frac{4}{5}$$

2. Solve the following inequalities.

a. $x^2 - 2x - 15 > 0$

$(x+3)(x-5) = 0$

$\therefore x = -3$ or $x = 5$

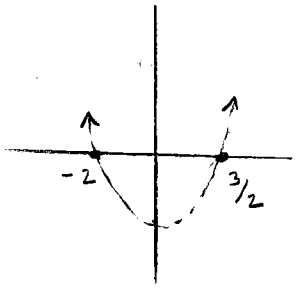



$\Rightarrow$ $x < -3$ or $5 < x$

b. $2x^2 + x - 6 < 0$

$(x+2)(2x-3) = 0$

$\therefore x = -2$ or $x = \frac{3}{2}$



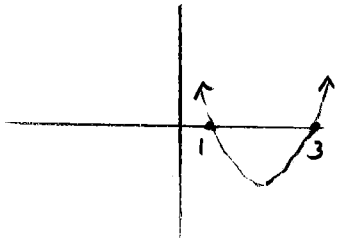

$\Rightarrow$ $-2 < x < \frac{3}{2}$

c. $-x^2 + 4x - 3 \geq 0$

$x^2 - 4x + 3 \leq 0$

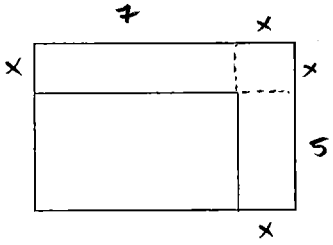
$(x-1)(x-3) = 0$

$\therefore x = 1$ or $x = 3$




$\Rightarrow$ $1 \leq x \leq 3$

3. A rectangular lawn measures 7 m by 5 m. A uniform border of flowers is to be planted along two adjacent sides of the lawn, as shown in the diagram. If the flowers that have been purchased will cover an area of 6.25 $m^2$, how wide is the border?




$A = lw$

$6.25 = x(7+x) + 5x$

$0 = 7x + x^2 + 5x - 6.25$

$0 = x^2 + 12x - 6.25$

$x = \frac{-12 \pm \sqrt{(12)^2 - 4(-6.25)}}{2}$

$= \frac{-12 \pm \sqrt{144 + 25}}{2}$

$x = \frac{-12 \pm \sqrt{169}}{2}$

$\Rightarrow$ $x_1 = 0.5$

$x_2 = -12.5$ (inadmiss.)

$\therefore$ the width of the border is 0.5 m.

4. The sum of the squares of three consecutive integers is 302. Find the integers.

Let $x$, $x+1$, $x+2$ be the integers.

$x^2 + (x+1)^2 + (x+2)^2 = 302$

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 302$

$3x^2 + 6x - 297 = 0$

$x^2 + 2x - 99 = 0$

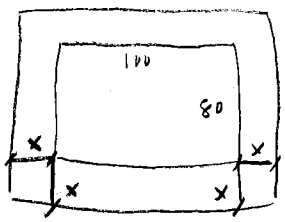
$\Rightarrow (x+11)(x-9) = 0$

$\therefore x = -11$ or $x = 9$

$\Rightarrow -10, 10$

$-9, 11$

5. A rectangular building 100 m by 80 m is to be surrounded by a lawn of uniform width. The area of the lawn must be equal to the area of the building. Find the width of the lawn to the nearest tenth of a metre.




$A_{lawn} = 8000$

$A_{tot} = A_{lawn} + A_{border}$

$A_{tot} = 16\,000$

$A_{tot} = (2x+100)(2x+80)$

$16\,000 = 4x^2 + 160x + 200x + 8000$

$0 = 4x^2 + 360x - 8000$

$= x^2 + 90x - 2000$

Using Quadratic Formula:

$x_1 = 18.4$ $\quad x_2 = -108.4$ (inadmissible)

6. Find the discriminant and determine the nature of the roots.

a. $2x^2 + x = 5 \Rightarrow 2x^2 + x - 5 = 0$

$b^2 - 4ac$

$= (1)^2 - 4(2)(-5)$

$= 1 + 40$

$= 41 \Rightarrow$ 2 Real, unequal Roots.

b. $5x^2 + 7x = 0$

$b^2 - 4ac$

$= (7)^2 - 4(5)(0)$

$= 49 \Rightarrow$ 2 Real, unequal Roots.

7. Determine the value of $k$ that will give the indicated solution.

a. $x^2 + (k-1)x + 1 = 0$, 2 unequal real roots

$b^2 - 4ac$

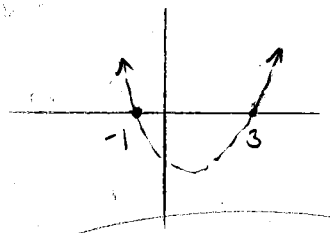
$(k-1)^2 - 4(1) > 0$

$k^2 - 2k + 1 - 4 > 0$

$k^2 - 2k - 3 > 0$

$(k-3)(k+1) = 0$

$\therefore k = 3 \; ; \; k = -1$




$k < -1$ OR $3 < k$

b. $(k+1)x^2 - 2x - 3 = 0$, 2 non real roots

$b^2 - 4ac < 0$

$(-2)^2 - 4(k+1)(-3) < 0$

$4 + 12k + 12 < 0$

$12k + 16 < 0$

$k < -\frac{16}{12}$ or $-\frac{4}{3}$

c. $x^2 + (k+2)x + 2k = 0$, 2 equal real roots

$b^2 - 4ac = 0$

$(k+2)^2 - 4(2k) = 0$

$k^2 + 4k + 4 - 8k = 0$

$k^2 - 4k + 4 = 0$

$(k-2)(k-2) = 0 \Rightarrow k = 2$

8. A sportswear store sells baseball caps with the local baseball team's logo on them. Last year, the store sold 600 caps at $15 each. The store manager is planning to increase the price. A consumer survey shows that for every $1 increase, there will be a drop of 30 sales a year.
   a. What should the selling price be to maximize the annual revenue?
   b. What is the maximum annual revenue from the caps?

$R = (15 + x)(600 - 30x)$

$= 9000 - 450x + 600x - 30x^2$

$= -30x^2 + 150x + 9000$

$= -30(x^2 - 5x) + 9000$

$= -30(x^2 - 5x + 6.25) + 187.5 + 9000$

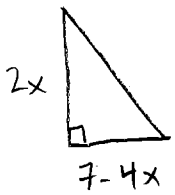
$= -30(x - 2.5)^2 + 9187.5$

$\therefore$ vertex $(2.5, 9187.5)$

a) selling price $= 15 + 1(2.5) = \$17.50$

b.) max. revenue $= \$9187.50$

9. A triangle has a height of $2x$ and a base of $7-4x$
   a. What is the maximum area of the triangle?
   b. What value of $x$ gives the maximum area?




$$A = \frac{2x(7-4x)}{2}$$

$$A = 7x - 4x^2$$

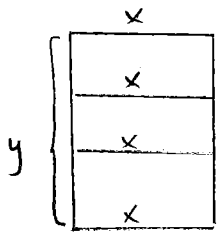
$$= -4x^2 + 7x$$

$$= -4(x^2 - 1.75x + 0.765625) + 3.0625$$

$$= -4(x - 0.875)^2 + 3.0625$$

∴ max area is 3.06 units$^2$ ; $x = 0.875$ unit

10. A rectangular field is to be enclosed by a fence. Two fences, parallel to one side of the field, divide the field into 3 rectangular fields. If 2400 m of fence are available, find the dimensions of the field giving the maximum area.




$$2400 = 4x + 2y$$

$$y = 1200 - 2x$$

$$A = xy$$

$$= x(1200 - 2x)$$

$$= 1200x - 2x^2$$

$$= -2x^2 + 1200x$$

$$= -2(x^2 - 600x)$$

$$= -2(x - 300)^2 + 180\,000$$

vertex $(300, 180\,000)$

∴ dimensions are 300 m × 600 m

max. area is 180 000 m$^2$

11. Solve each system of equations.

a) $y - 9 = 0$
$y - x^2 = 0$

$y = 9$

$y = x^2$

$9 = x^2$

$x = \pm 3$

Subst. $x = 3$:

$y = (3)^2$

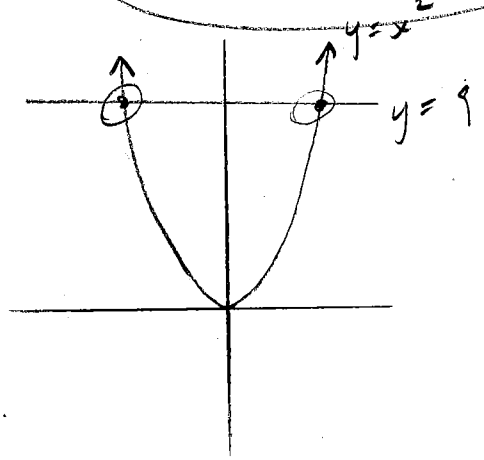
$y = 9$

Subst. $x = -3$:

$y = (-3)^2$

$y = 9$.

$\therefore (3, 9)$ and $(-3, 9)$.




b) $y = -2x^2 + 9x + 9$
$y = -3x - 5$

$-3x - 5 = -2x^2 + 9x + 9$

$2x^2 - 12x - 14 = 0$.

$2(x^2 - 6x - 7) = 0$.

$(x - 7)(x + 1) = 0$

$\therefore x = 7$ or $x = -1$

Subst. $x = 7$ into ②:

$y = -3(7) - 5$

$y = -26$

Subst. $x = -1$ into ②:

$y = -3(-1) - 5$

$y = -2$.

$\therefore (x, y) = (7, -26)$

and

$(x, y) = (-1, -2)$.